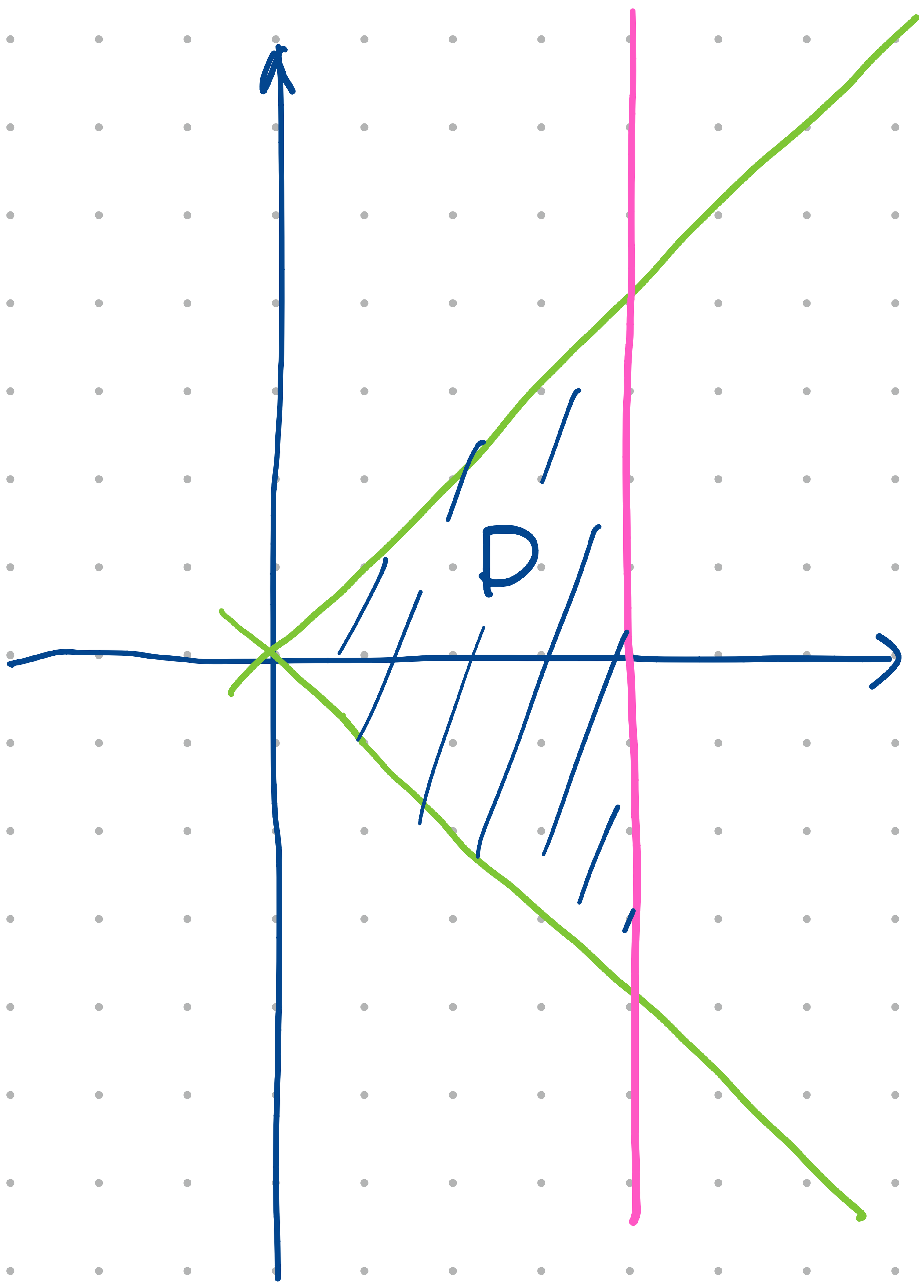
Compute the following integral by converting to Cartesian.

$$\int_{-\pi/4}^{\pi/4} \int_0^{\sec\theta} r^4 \cos\theta \, dr\, d\theta$$



$x = r\cos\theta$

$y = r\sin\theta$

integral bounds:

$$\boxed{-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}}$$

$0 \le r \le \sec\theta$

What is $\boxed{r = \sec\theta}$?

$r\cos\theta = 1$

i.e. $x = 1$

$$r^4 \cos\theta \, dr\, d\theta$$

$$= r^3 \cos\theta \, r\, dr\, d\theta$$

$$= (r^2)(r\cos\theta) \, r\, dr\, d\theta$$

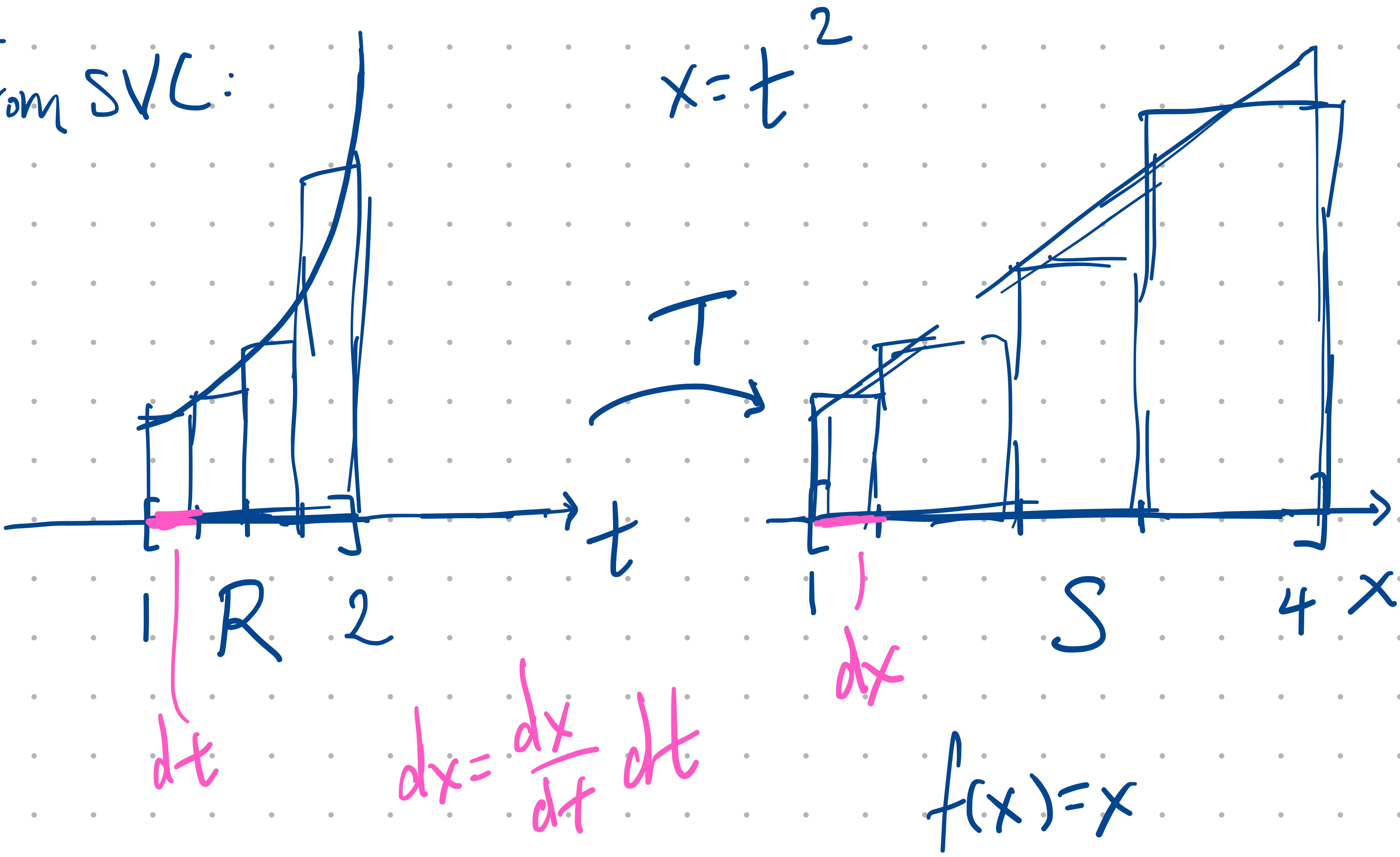
$$\iint_D r^4 \cos\theta \, dr\, d\theta = \iint_D (x^2+y^2)\, x \, dy\, dx$$

$$= \int_0^1 \int_{-x}^{x} (x^3 + xy^2)\, dy\, dx$$

$$= \ldots\ldots\ldots$$

$$= 8/15$$

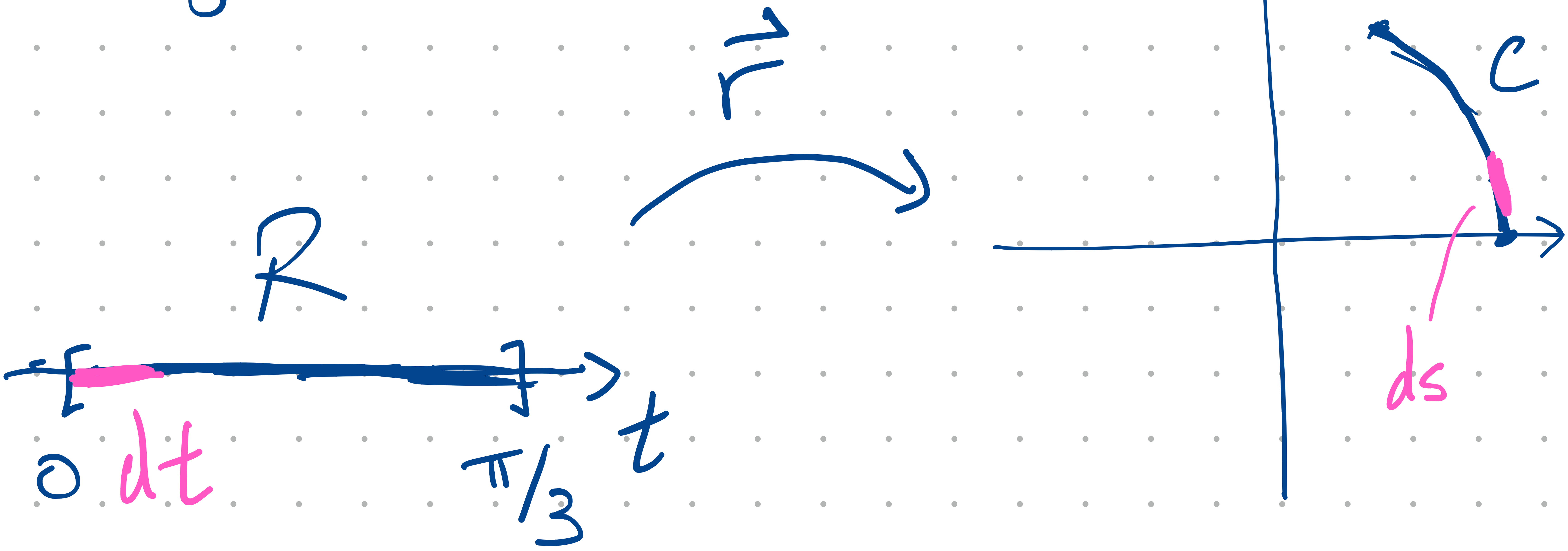
From SVC:



The transformation T allows for the conversion of an integral on S to an integral on R, keeping in mind that T alters measurements in the region of integration.

$$\int_R f(t^2)\,\frac{dx}{dt}\,dt = \int_S f(x)\,dx$$

Earlier in this course, encountered an arclength formula.



$$\vec{r}(t) = \langle 2\cos t,\ 2\sin t \rangle$$

Arclength:

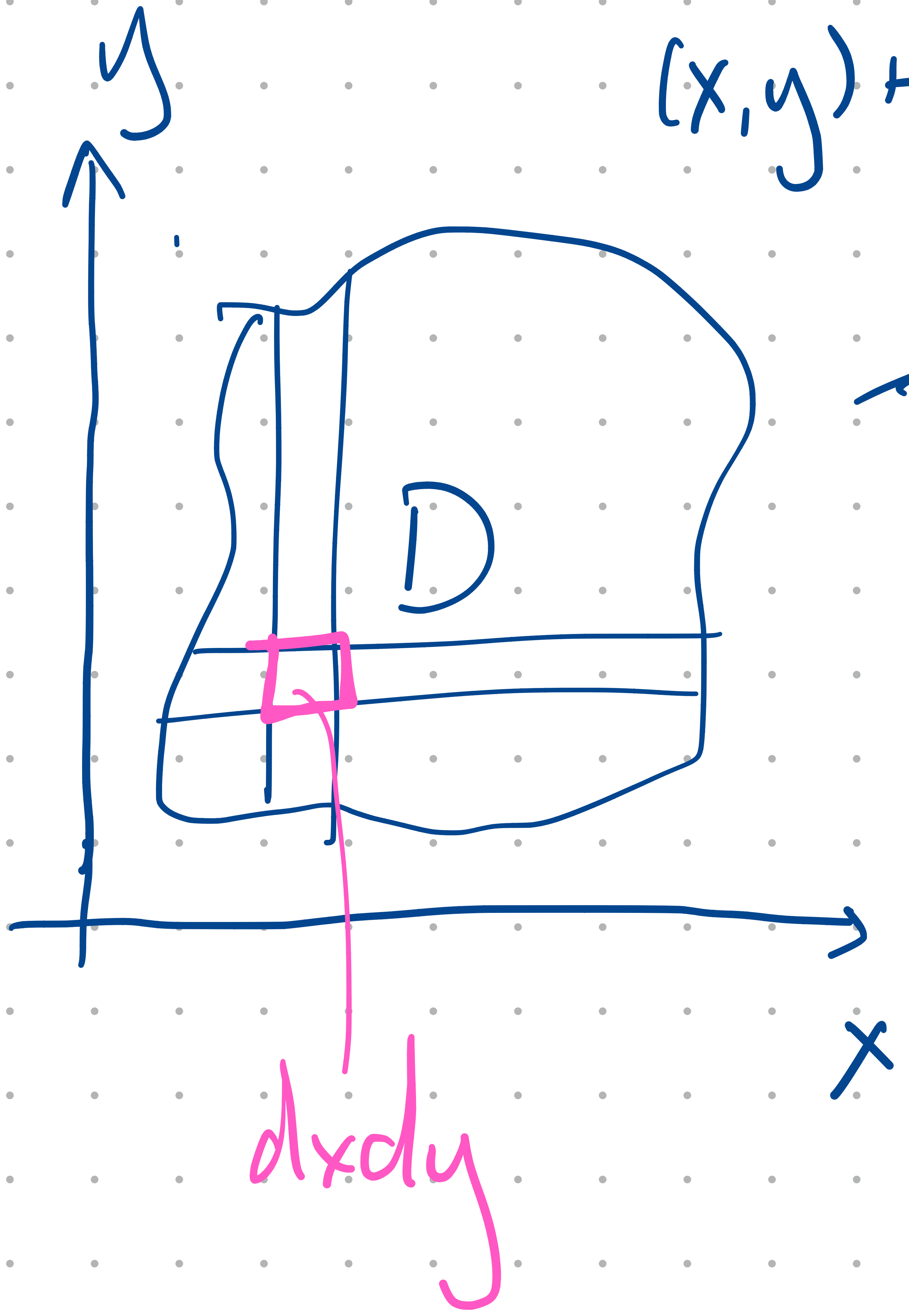
$$\int_R 1 \boxed{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}\, dt = \int_C 1\, ds$$

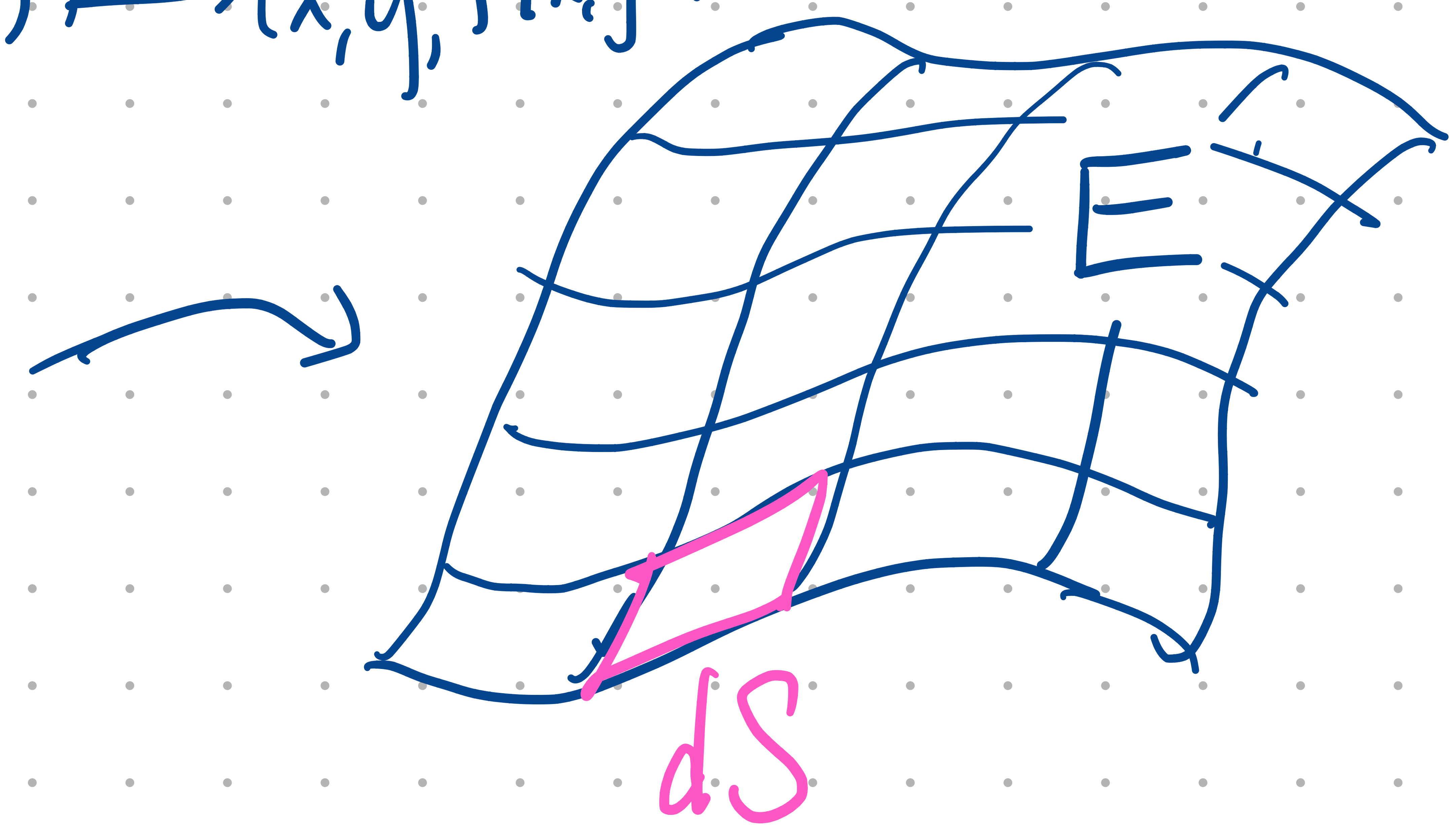
How does dt relate to ds? $ds = \frac{ds}{dt}\, dt$

rate of change of distance, i.e. speed.

Speed $= |\vec{r}\,'(t)|$

$(x,y) \longmapsto (x,y,f(x,y))$





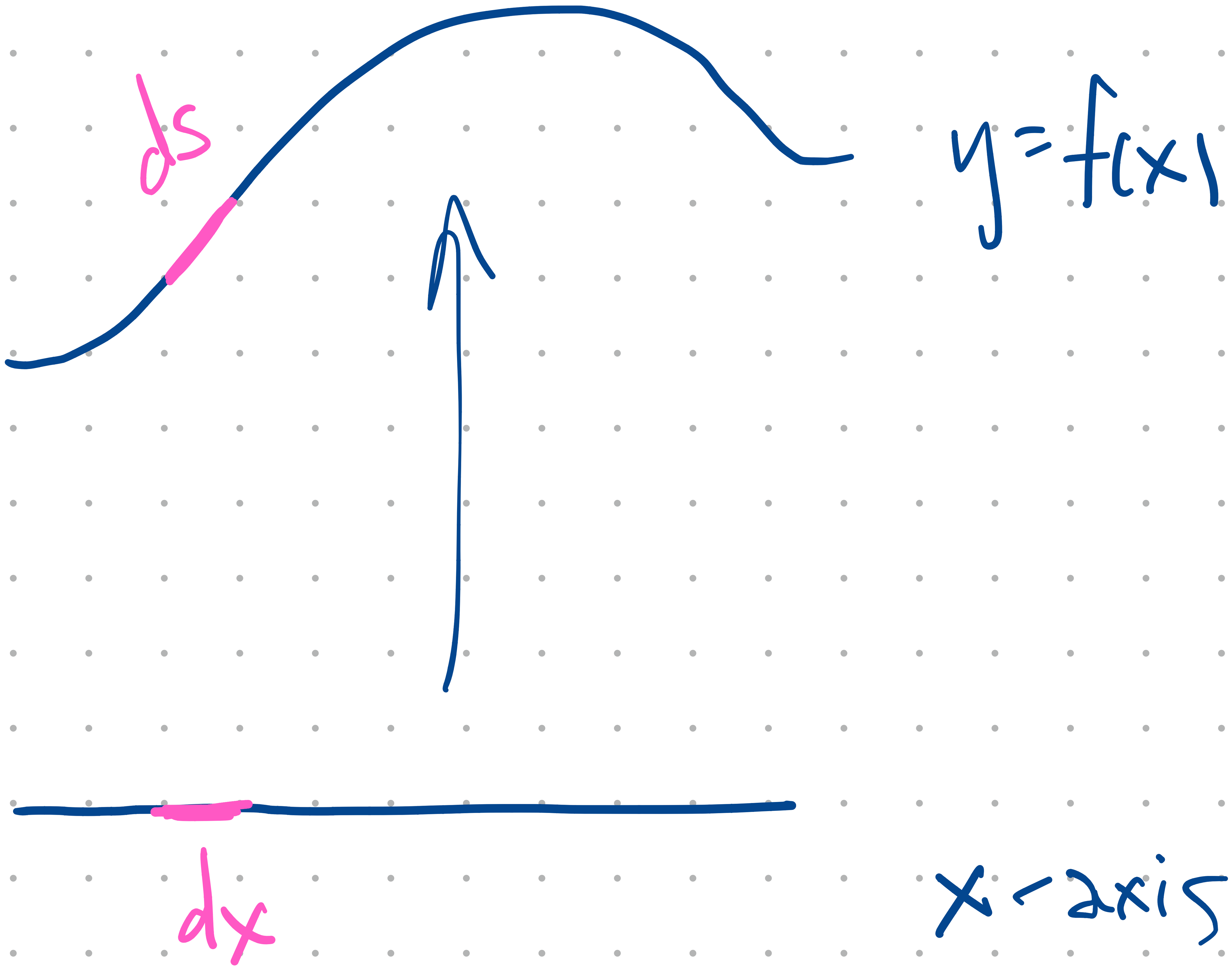
Surface area:

$$\iint_D 1\sqrt{f_x^2+f_y^2+1}\,dxdy = \iint_E 1\,dS$$

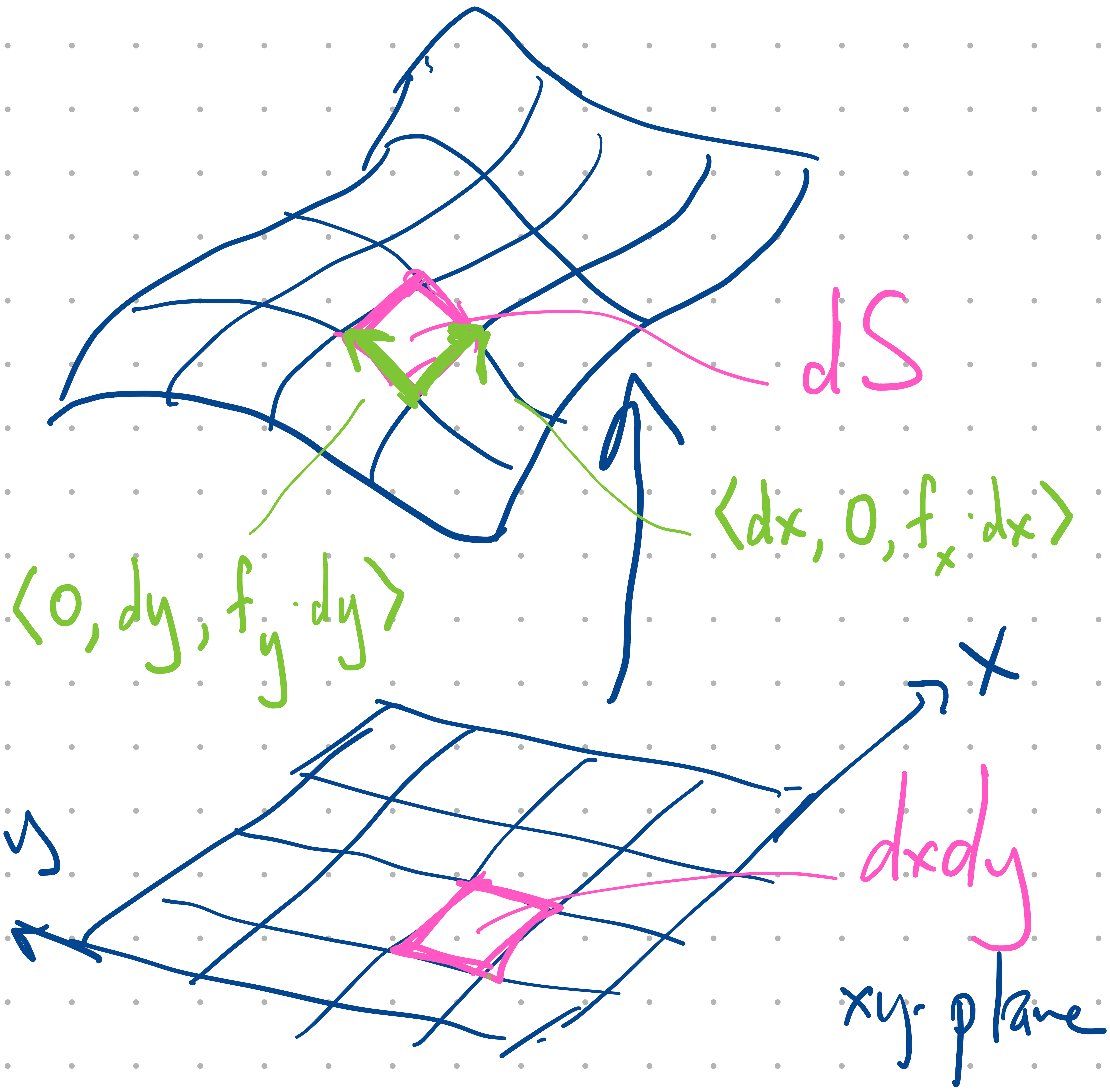
$$dS = \left|\det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & f_x dx \\ 0 & dy & f_y dy \end{bmatrix}\right| = \left|\langle -f_x\,dxdy,\ -f_y\,dxdy,\ dxdy\rangle\right|$$

$$= \left|\langle -f_x, -f_y, 1\rangle\right| dxdy$$

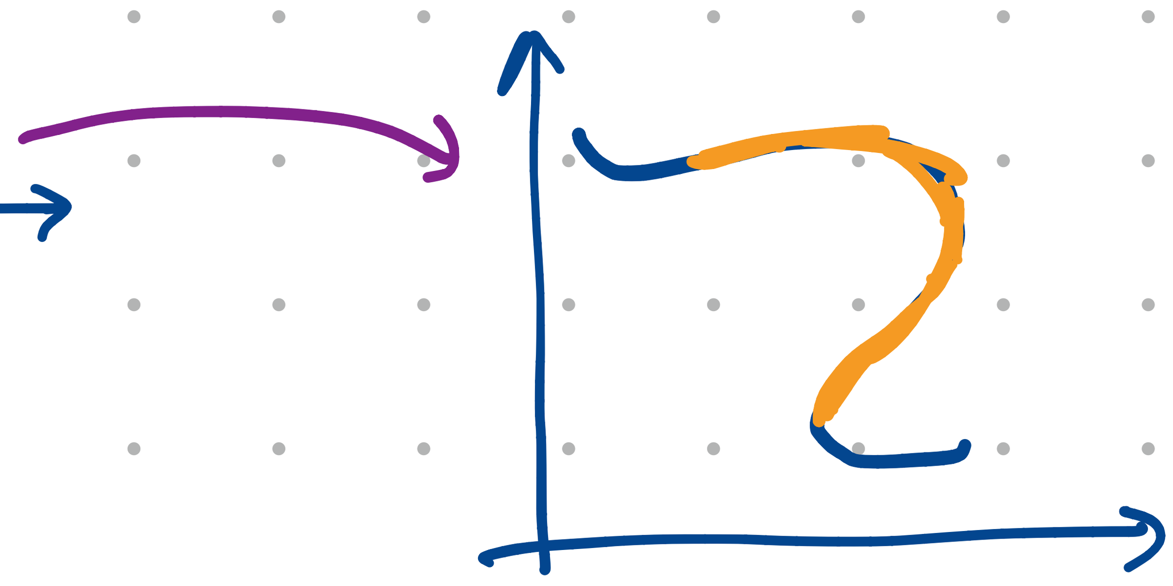
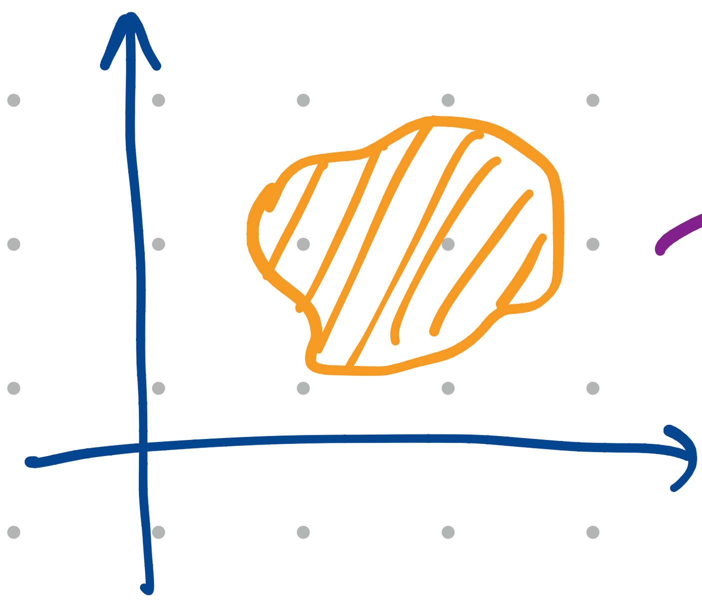
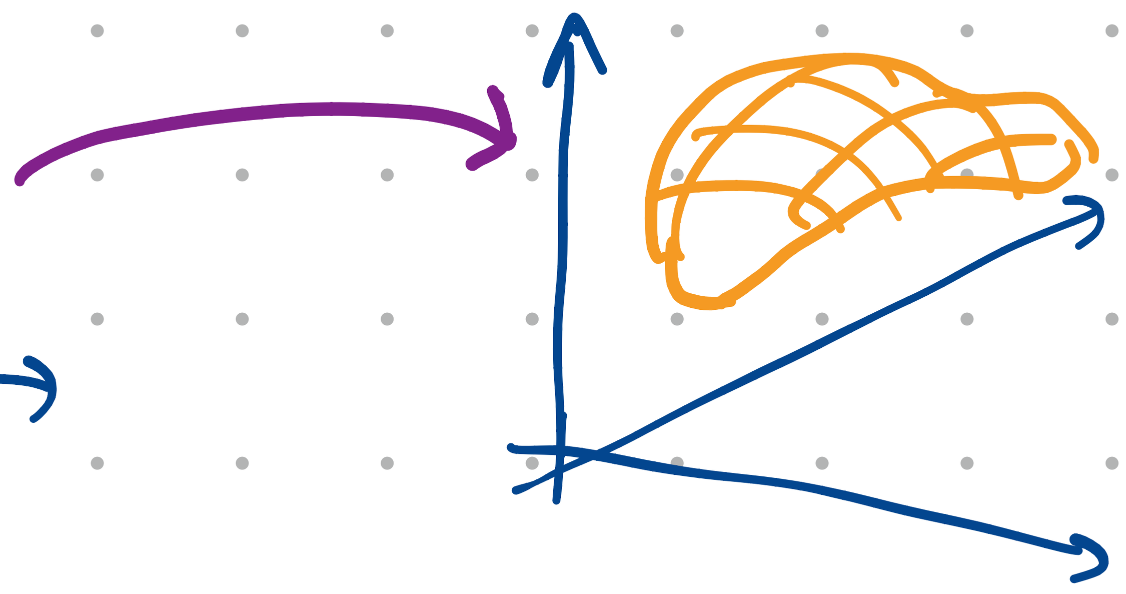
$$= \sqrt{f_x^2+f_y^2+1}\ dxdy$$



ds
y=f(x)
dx
x-axis



dS
⟨dx, 0, f_x · dx⟩
⟨0, dy, f_y · dy⟩
x
y
dxdy
xy-plane

	in 1D	in 2D	in 3D
1D region			
2D region	N/A		
3D region	N/A	N/A